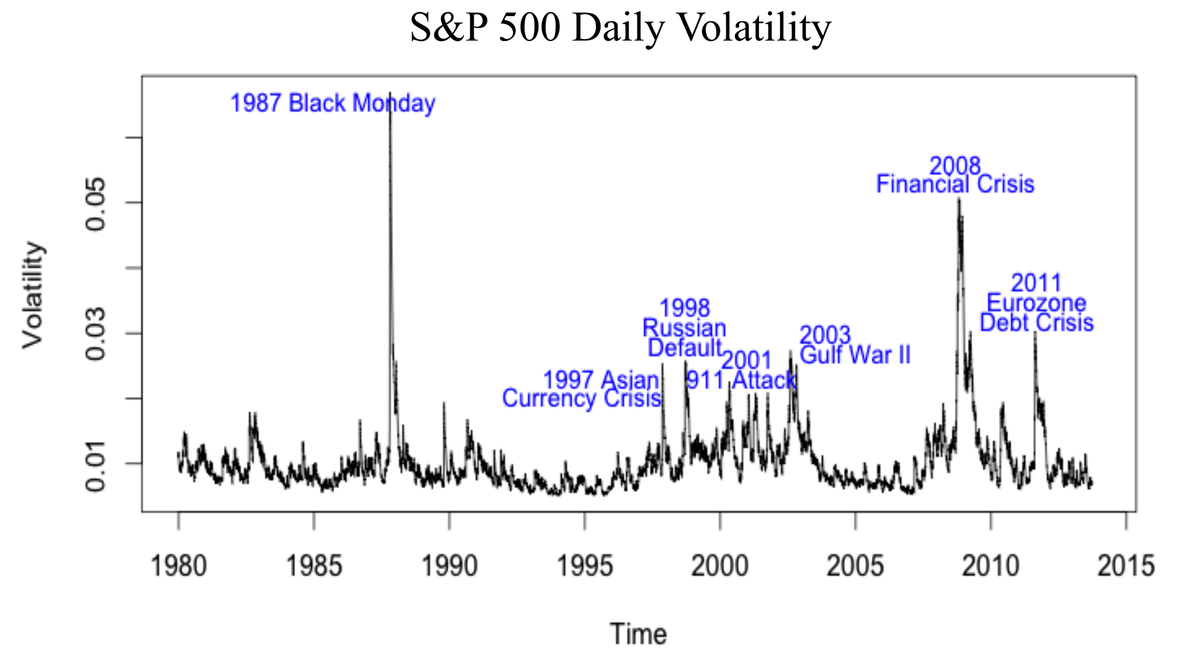
New Development of Volatility Inference in Financial Market: Usage of High-frequency Financial Data

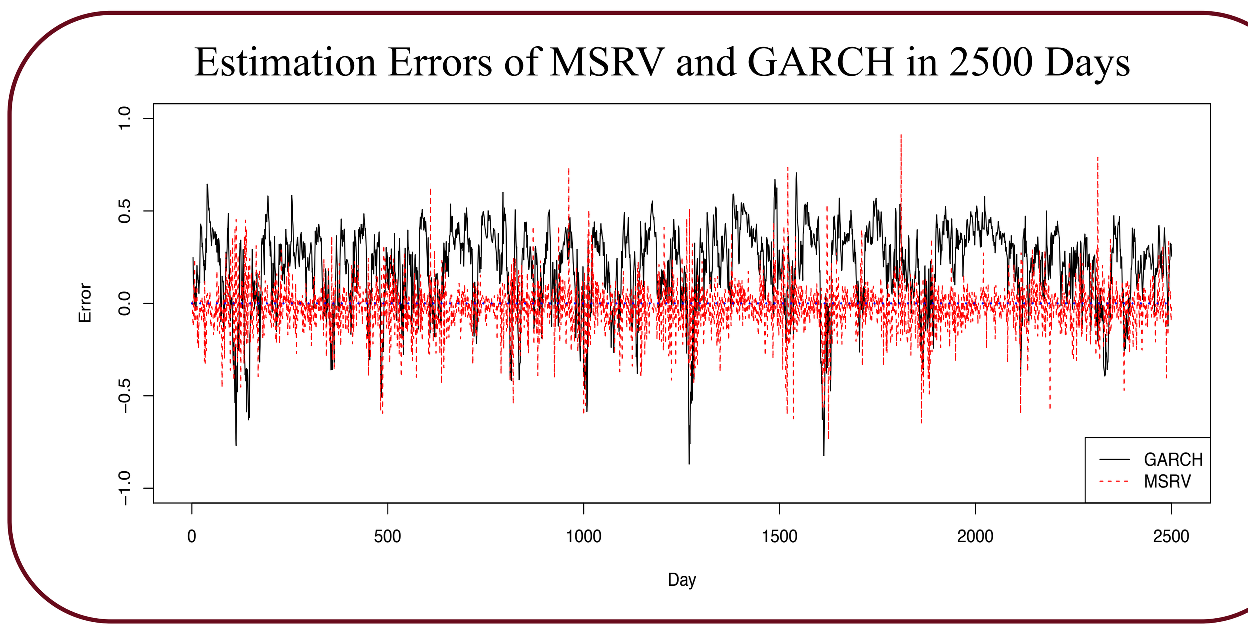
Introduction

- Volatilities, reflecting market risk, are crucial important in portfolio allocation, decision-making, and performance evaluation, etc.
- High-frequency financial data provide opportunity to analyze the dynamic of financial markets, in particular, the market volatility.



Results

Comparison: performance of high-frequency estimator (MSRV) and low-frequency estimator (GARCH)



- GARCH model over-estimates volatilities in most of the days.
- Our method, MSRV, produces smaller estimation errors; is better than GARCH in capturing the dynamics of true volatility process.

Methodology

***K* Groups and Sub-Sampling**

(help to control the effect of noise)

$\boldsymbol{\tau}^1: t_1, t_{K+1}, t_{2K+1}, \cdots$

$\boldsymbol{\tau}^2: t_2, t_{K+2}, t_{2K+2}, \cdots$

$\cdots$

$\boldsymbol{\tau}^k: t_k, t_{K+k}, t_{2K+k}, \cdots$

$\cdots$

$\boldsymbol{\tau}^K: t_K, t_{2K}, t_{3K}, \cdots$

Construct Co-Volatility and One-Scale Volatility Matrix

Co-volatility: $\tilde{\Gamma}_{ij}(\boldsymbol{\tau}^k) = \sum_{r=2}^{|\boldsymbol{\tau}^k|}\left[Y_i(\tau_r^k) - Y_i(\tau_{r-1}^k)\right]\left[Y_j(\tau_r^k) - Y_j(\tau_{r-1}^k)\right]$

One-scale volatility: $\tilde{\Gamma}_{ij}^K = \frac{1}{K}\sum_{k=1}^{K}\tilde{\Gamma}_{ij}(\boldsymbol{\tau}^k)$, and $\tilde{\Gamma}^K = \left(\tilde{\Gamma}_{ij}^K\right)$

Define Multi-Scale Realized Volatility (MSRV) Matrix Estimator

(This MSRV estimator will minimize the effect of noise)

$$\tilde{\Gamma} = \sum_{m=1}^{N} a_m \tilde{\Gamma}^{K_m} + \zeta\left(\tilde{\Gamma}^{K_1} - \tilde{\Gamma}^{K_N}\right),\ a_m = \frac{12K_m(m - N/2 - 1/2)}{N(N^2-1)},\ \zeta = \frac{K_1 K_N}{n(N-1)}$$

Good for fixed (or small) number of assets

Extra work for large number of assets

Impose Sparsity Condition for Large Volatility Matrix Γ

$$\sum_{j=1}^{p}\left|\Gamma_{ij}\right|^q \le \Psi \pi_n(p), \text{ for any } i = 1, \cdots, p$$

Define Threshold MSRVM Estimator

(solve the high-dimensional problem)

$$\hat{\Gamma} = \left(\tilde{\Gamma}_{ij} 1\left(\left|\tilde{\Gamma}_{ij}\right| \ge \varpi\right)\right)$$

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